

Module 2: Introduction to Statistics

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Topic

- Dichotomous Variables
- Compare Proportions
 - Two sample test (Normal approximation theory)
 - Chi-square test
 - Fisher Exact test
- Measuring Treatment Effect on Binary Outcomes
 - Absolute Risk Reduction (ARR)
 - Relative Risk (RR)
 - Odds Ratio (OR)
- Application and Discussion of a Research Article
 - Feasibility of treating prehypertension with an angiotensin-receptor blocker. Julius S. *et al.* *N Engl J Med.* 2006; 354:1685-97

Dichotomous Variables: Binary Data

- Binary variables indicate two different states
 - Presence or absence of a characteristic: $X=1$ (Yes)/ 0 (No)
 - Tossing a Coin: $Pr(Tail)=0.5$
 - $Pr(\text{Carrying Gene G})=p$
$$X_i \sim \text{Bernoulli}(p)$$
 - Choose a cutoff point in continuous measure
 - Obesity: $BMI \geq 30$ kg/m²
 - Hypertension: $SBP \geq 140$ or $DBP \geq 90$ mmHg
 - Assign status based on a checklist
 - Depressed: (If 16 or more items from the checklist are checked)
 - Control: (If < 16 items from the checklist are checked)

Binomial Distribution

- Y is the number of successes in a fixed number (n) of independent Bernoulli trials (X_i) with the same probability of success in each trial
 - $X_i \sim \text{Bernoulli}(p)$
 - $Y = \sum_{i=1}^n X_i$
$$Y \sim \text{Bin}(n, p)$$
- Requirements
 1. Each trial has one of two possible outcomes (1=success/0=fail)
 2. The trials are independent
 3. Probability of success (event) is the same in all trials
 4. A fixed number of trials (i.e. $n=100$)

Mean and Standard Deviation of Number of Successes: $Y \sim \text{Bin}(n,p)$

- Mean of Y:
 - If a coin is tossed $n=100$, what is the expected number of Tails?

$$E(Y)=np=50$$

- n is the number of trials
- p is the probability of success
- Variance and Standard Deviation:

$$\text{Var}(Y)=np(1-p)=100 \times 0.5 \times 0.5=25$$

$$\text{SD}(Y)=\sqrt{np(1-p)}$$

Mean and Standard Deviation of Proportion $\bar{Y} \sim \text{Bin}(n,p)$

- Estimate of Proportion:
 - If an unfair coin is tossed 100 times and the result is 25 Tails, what is the expected value of p ?

$$\hat{p} = \frac{Y}{n} = \bar{Y} = \frac{25}{100} = .25$$

$$E(\bar{Y})=p$$

- Y number of successes
- n number of trials
- p probability of success
- Variance and Standard Deviation of \bar{Y} :

$$\text{Var}(\bar{Y})= p(1-p)/n \approx \hat{p}(1 - \hat{p})/100$$

$$\text{SD}(\bar{Y})=\sqrt{p(1-p)/n}$$

Which of These Variables Would Have a Binomial Distribution?

- Number of female students in this class given the total number of students
 - ✓ Yes
- BMI of 100 people
 - X No
- Number of people with BMI ≥ 30 kg/m²
 - ✓ Yes

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Examples of Testing for Differences Between Two Proportions

- Does the proportion of patients with hypertension differ between two groups?
 - Treatment vs. Control
 - Smoker vs. Non smoker

Notation and Display of Categorical Data 2 x 2 Contingency Tables

	Hypertension		Total
	Yes	No	
Treatment	n_{11}	n_{12}	$n_{1.}$
Placebo	n_{21}	n_{22}	$n_{2.}$
Total	$n_{.1}$	$n_{.2}$	n

n_{ij} are referred to as cell frequencies.
 $n_{.j}$ and $n_{i.}$ are referred to as marginal frequencies
 n is the total sample size

Example: 2 x 2 Tables

TROPHY data	Hypertension		Total
	Yes (% of row)	No	
Treatment	14(11%)	113	127
Placebo	57(44.5%)	71	128
Total	71(27.8%)	184	255

Proportion of HT in Treatment group: $p_1 = 14/127 = 11\%$

Proportion of HT at Placebo group: $p_2 = 57/128 = 44.5\%$

Proportion of HT in both groups: $p = 71/255 = 27.8\%$

Q: What is the number of subjects with HT from the Treated group?

Test for Differences in Proportions Between Two Groups

- Testing whether the proportions for some outcome (e.g. HT) are different between two groups:

$$H_0: p_1 = p_2$$

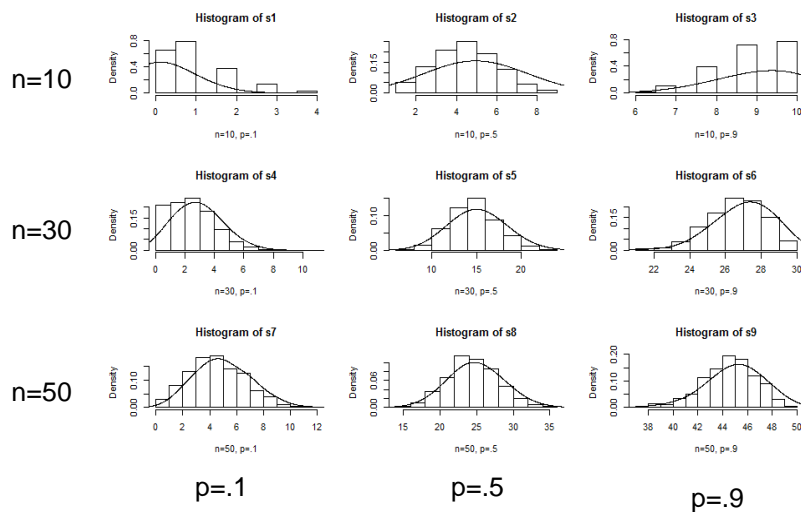
vs.

$$H_A: p_1 \neq p_2$$

Three Tests for Differences in Proportions Between Two Groups

- Two-sample test for differences in two proportions
 - Normal theory test, works for large n due to CLT
- $$Y = \sum_{i=1}^n X_i$$
- Chi-Square test
 - Works when $n > 5$ in all cells
 - Fisher's Exact test
 - Works for any n , but computationally intensive when n is large
 - Used when n is not large, otherwise use the Chi-Square test

Normal theory test: $Y \sim \text{Bin}(n, p)$ is approximate normal for large n (CLT)



Test Statistics for Difference in Two Binomial Proportions (Normal theory test)

\hat{p}_1 : proportion in group 1 with outcome (sample size is n_1)

\hat{p}_2 : proportion in group 2 with outcome (sample size is n_2)

\hat{p} : Overall proportion for group 1 and 2 combined

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Can be used only if

$$n_1\hat{p}_1(1 - \hat{p}_1) > 5$$

$$n_2\hat{p}_2(1 - \hat{p}_2) > 5$$

e.g. $p=.5$ and $n > 20$

$p=.1$ and $n > 56$

TROPHY Data test for Binomial Proportions (Normal theory test)

TROPHY data	Hypertension		Total
	Yes (% of row)	No	
Treatment	14(11%)	113	127 (n_1)
Placebo	57(44.5%)	71	128 (n_2)
Total	71(27.8%)	184	255

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p}_1 = 14/127 = 11\%$$

$$\hat{p}_2 = 57/128 = 44.5\%$$

$$\hat{p} = 71/255 = 27.8\%$$

TROPHY Data test for Binomial Proportions (Normal theory test)

$$z = \frac{.11 - .445}{\sqrt{.278 * (1 - .278) \left(\frac{1}{127} + \frac{1}{128} \right)}} = \frac{-.335}{\sqrt{.207 * .01569}} = -5.96$$

p-value = 2.52×10^{-9} , Reject $H_0: p_1 = p_2$

Chi-Square (χ^2) Test

The Chi-Square test is the most commonly used test for categorical data analysis

- Can be used for 2 x 2 tables
- Can be used for n x m tables (for any n and m)

Observed Cell Proportions (Deriving χ^2 Test)

	Hypertension		Total
	Yes	No	
Treatment	14(5.5%)	113(44.3%)	127(49.8%)
Placebo	57(22.4%)	71(27.8%)	128(50.2%)
Total	71(27.8%)	184(72.2%)	255

Cell % relative to the overall n=255

E.g. What proportion of the total sample is from the treatment group and has HT?

$$14/255 = 5.5\%$$

Expected Cell Proportions (Deriving χ^2 Test)

TROPHY data	Hypertension		Total
	Yes	No	
Treatment	14	113	127(49.8%)
Placebo	57	71	128(50.2%)
Total	71(27.8%)	184(72.2%)	255

Marginal Proportions:

- Marginal Row %: What proportion is in the Treatment (Placebo) group?
 $127/255 = 49.2\%$
- Marginal Column %: What proportion is HT (Not HT)?
 $71/255 = 27.8\%$

Expected Cell Proportions (Deriving χ^2 Test)

TROPHY Data	Hypertension		Total
	Yes	No	
Treatment	13.8%	36%	49.8%
Placebo	14%	36.2%	50.2%
Total	27.8%	72.2%	100%

Marginal proportion are fixed.

Q: What proportion of the total sample is expected in each cell (when H_0 is true)?

Multiply the row percent with column percent:

$$27.8\% \times 49.8\% = 13.8\%$$

Expected Cell Frequency (Deriving χ^2 Test)

TROPHY Data	Hypertension		Total
	Yes	No	
Treatment	35.2(13.8%)	91.8	127
Placebo	35.7	92.3	128
Total	71	184	255

What number from the total sample is expected in each cell?

$$13.8\% \times 255 = 35.2$$

Compare Observed vs. Expected Frequencies (Deriving χ^2 Test)

TROPHY Data	Hypertension		Total
	Yes	No	
Treatment	14/35.2	113/91.8	127
Placebo	57/35.7	71/92.3	128
Total	71	184	255

Observed frequencies: $O_{11} = 14$

Expected frequency: $E_{11} = 35.2$

If H_0 is true then O_{11} should be close to E_{11}

Chi-Square Test

- Chi-Square test, with Yate's correction, is based on:

$$\chi^2 = \frac{(|O_{11} - E_{11}| - .5)^2}{E_{11}} + \frac{(|O_{12} - E_{12}| - .5)^2}{E_{12}} + \frac{(|O_{21} - E_{21}| - .5)^2}{E_{21}} + \frac{(|O_{22} - E_{22}| - .5)^2}{E_{22}}$$

- χ^2 has a Chi-Square distribution with $df = k(?)$
- Calculate the p-value based on the Chi-Square distribution with k df
 - If p-value < 0.05 reject H_0

Chi-Square Test: Calculating Degrees of Freedom

TROPHY Data	Hypertension		Total
	Yes	No	
Treatment	14		127
Placebo			128
Total	71	184	255

For 2 x 2 tables, the frequency number in only one cell is free to vary. Frequencies in the remaining 3 cell are constrained and can be derived.

What is the frequency for non HT in the Treated group?

Chi-Square Test: Calculating Degrees of Freedom

TROPHY Data	Hypertension		Total
	Yes	No	
Treatment	14	113(127-14)	127
Placebo	57 (71-14)	71(128-57)	128
Total	71	184	255

- $df = (\text{Rows}-1) \times (\text{Columns}-1) = 1$
- Then, use the Chi-Square with 1 df to derive the p-value.
If p-value < .05, then reject $H_0: p_1 = p_2$

Chi-Square Test in R

- In R: `chisq.test(HT,Trt)`

- Output:

Pearson's Chi-squared test with Yates' continuity correction

data: HT and Trt

X-squared = 33.9775, df = 1, p-value = 5.575e-09 → Reject H_0 of no treatment effect

Fisher's Exact Test

- Fisher's exact test is not based on the normal approximation theory. It is an exact test
- It calculates the exact probability (under H_0) that one would observe a 2 x 2 table same or more extreme than the one observed (if $< .05$ reject H_0)
- It is used when n is small, and the Chi-square test or the normal approximation theory might not apply

Example: 2 x 2 Contingency Table Fisher's Exact Test (Small Sample)

Example	Not HT	HT	Total
Treated	4	0	4
Placebo	1	3	4
Total	5	3	8

Marginal counts (are fixed)

- Under the H_0 of no difference on HT between two groups, calculate the probability of each table with the same marginal counts
- How many Tables with these given margins are possible?

Example	Not HT	HT	Total
Treated	?		4
Placebo			4
Total	5	3	8

All Tables With Same Marginal Counts

Table 1	No HT	HT	Total
Treated	4		4
Placebo			4
Total	5	3	8

Table 2	No HT	HT	Total
Treated	3		4
Placebo			4
Total	5	3	8

Table 3	No HT	HT	Total
Treated	2		4
Placebo			4
Total	5	3	8

Table 4	No HT	HT	Total
Treated	1		4
Placebo			4
Total	5	3	8

Table 5	No HT	HT	Total
Treated	0		4
Placebo	5(?)		4
Total	5	3	8

All Tables With Same Marginal Counts

Table 1	No HT	HT	Total
Treated	4	0	4
Placebo	1	3	4
Total	5	3	8

Table 2	No HT	HT	Total
Treated	3	1	4
Placebo	2	2	4
Total	5	3	8

Table 3	No HT	HT	Total
Treated	2	2	4
Placebo	3	1	4
Total	5	3	8

Table 4	No HT	HT	Total
Treated	1	3	4
Placebo	4	0	4
Total	5	3	8

Tables (1 and 4) are same or less likely than the observed data (Table 1)

Total Probabilities: Table 1 = 0.071
 Table 2 = 0.429
 Table 3 = 0.429
 Table 4 = 0.071

The p-value for Fisher exact test is: $p=0.071+0.071=.142$

Table1: How Many Combinations Can Have This Result?

Table 1	No HT	HT	Total
Treated	4	0	4(A,B,C,D)
Placebo	1	3	4(a,b,c,d)
Total	5	3	8

Treatment row: 1 combination
 Placebo row: 4 combinations

Total: $1 \times 4 = 4$ Tables

Table 1a	No HT	HT	Total
Treated	4 (A,B,C,D)	0	4
Placebo	1 (a)	3 (b,c,d)	4
Total	5	3	8

Table 1b	No HT	HT	Total
Treated	4 (A,B,C,D)	0	4
Placebo	1 (b)	3 (a,c,d)	4
Total	5	3	8

Table 1d	No HT	HT	Total
Treated	4 (A,B,C,D)	0	4
Placebo	1 (d)	3 (a,b,c)	4
Total	5	3	8

Table 1c	No HT	HT	Total
Treated	4 (A,B,C,D)		4
Placebo	1 (c)	3 (a,b,d)	4
Total	5	3	8

How Many Total Tables are Possible?

Table 1	Not HT	HT	# Tables	Proportion
Treatment	4	0	$1*4=4$	$4/56=.071$
Placebo	1	3		
Table 2				
Treatment	3	1	$4*6=24$	$24/56=.429$
Placebo	2	2		
Table 3				
Treatment	2	2	$6*4=24$	$24/56=.429$
Placebo	3	1		
Table 4				
Treatment	1	3	$4*1=4$	$4/56=.071$
Placebo	4	0		
Total			56	1.00

Fisher's Exact Test in R

- In R: `fisher.test(HT,Trt)`

- R output:

Fisher's Exact Test for Count Data

data: HT and Trt

p-value = 0.1429

alternative hypothesis: true odds ratio is not equal to 1

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How to Measure Treatment Effect for Binary Data

There are several measures of a treatment effect (or associations) for binary data. Three most commonly used are:

- Absolute Risk Reduction (ARR)
- Relative Risk Reduction (RR)
- Odds Ratio (OR)

Absolute Risk Reduction (ARR)

TROPHY data	Hypertension		Total
	Yes (% of row)	No	
Treatment	14(11%)	113	127
Placebo	57(44.5%)	71	128
Total	71(27.8%)	184	255

- Risk of HT is measured by the probability of developing HT: $Pr(HT=Yes)$.

$$Pr(HT=Yes|Treated)=11\%$$

$$Pr(HT=Yes|Placebo)=44.5\%$$

- Absolute risk reduction (ARR) measures how much the risk is reduced due to Treatment?

$$ARR=44.5\% - 11\%=33.5\%$$

- If $ARR=0$, no Trt effect

Relative Risk (RR)

TROPHY data	Hypertension		Total
	Yes (% of row)	No	
Treatment	14(11%)	113	127
Placebo	57(44.5%)	71	128
Total	71(27.8%)	184	255

- Relative risk (RR) measures how much the risk is reduced due to Treatment relative to Placebo?

$$RR = \frac{0.11}{0.445} = 0.25$$

- If $RR=1$, no Trt effect

Which is a Better Measure: ARR or RR?

- The ARR and RR are sensitive to the magnitude of the proportions:

Ex 1: ARR=2%-1%=1% (small effect)
RR=1%/2%=0.5 (big effect)

Ex 2: ARR=95%-80%=15% (big effect)
RR=.95/.8=0.84 (small effect)

- Always report both the ARR and the RR

Odds Ratio(OR)

TROPHY data	Hypertension		Total
	Yes (% of row)	No	
Treatment	14(11%)	113	127
Placebo	57(44.5%)	71	128
Total	71(27.8%)	184	255

- Odds of developing HT are: $ODD = \frac{\Pr(HT=Yes)}{\Pr(HT=No)} = p/1-p$

$$ODD(Treated) = .11/.89 = .124 \quad ODD(Placebo) = .445/.556 = .80$$

- Odds Ratio (OR) measures how much the Odds are reduced due to Treatment compared to Placebo.

$$OR = \frac{.124}{.80} = 0.16 \quad (\text{if } OR=1, \text{ no Trt effect})$$

Odds Ratio(OR)

- OR are useful for measuring the relationship of any variable (Age, Trt) with a binary outcome (HT). They are usually derived using logistic regression
- In short, logistic regression is a statistical modeling technique used to predict the ODDs of HT (or any binary outcome) based on one or more variables

Modeling OR (log-OR) as a function of other predictors

- Logistic regression model is:

$$\log\left(\frac{\Pr(HT=1)}{1-\Pr(HT=1)}\right) = \beta_0 + \beta_1 * \text{Trt} + \beta_2 * \text{BMI} + \beta_3 * X + \dots$$

- $\text{OR}(\text{Trt}) = e^{\beta_1}$
Compares the ODDs of HT between Treatment and Placebo
- $\text{OR}(\text{BMI}) = e^{\beta_2}$
How much the ODDs of HT change if BMI increases by 1 (e.g. BMI=27 vs. BMI=26)
- $\text{OR}(X) = e^{\beta_3} = 1$, implies no relationship between X and Y.

Q: If X does not relate to Y, what is β_3 ?

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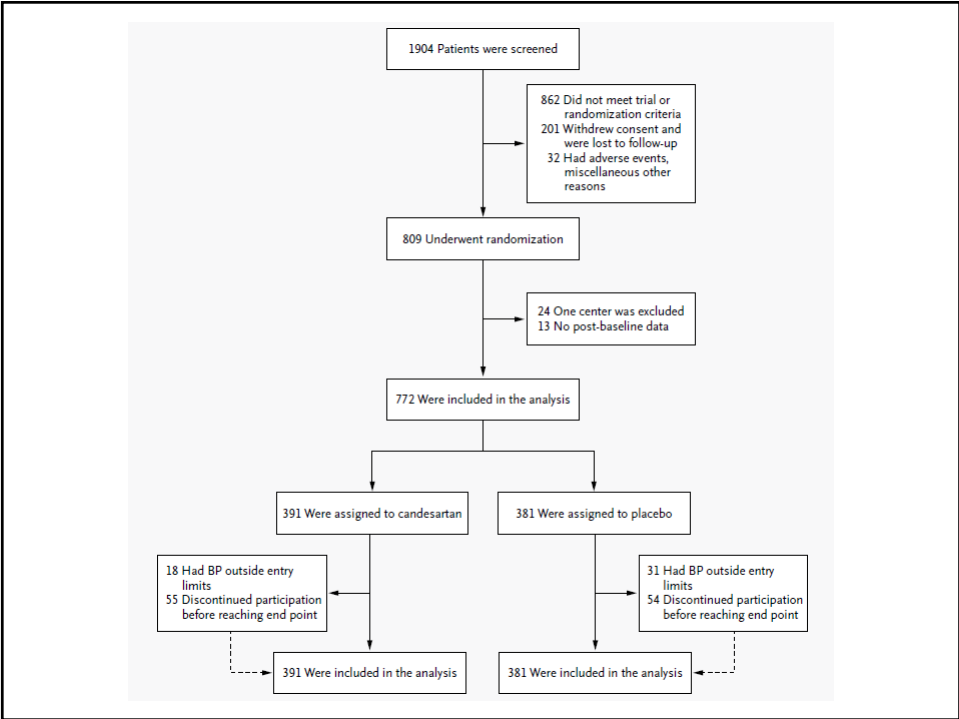
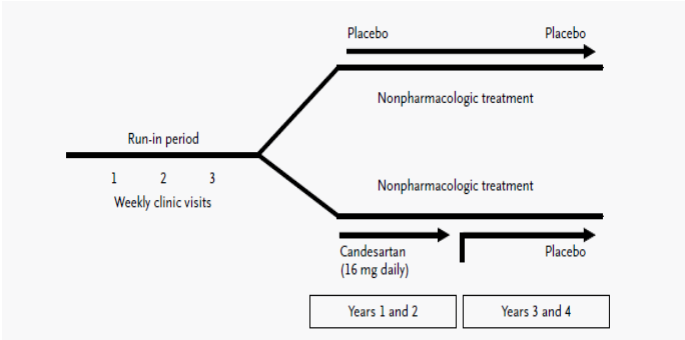
Application and Discussion of a Research Article*

- **Trial of Preventing Hypertension (TROPHY Study)**
 - **Background:** Hypertension is a strong predictor of excessive cardiovascular risk. TROPHY study investigated whether pharmacologic treatment of prehypertension prevents or postpones hypertension, thus reducing the CV risk.

*Feasibility of treating prehypertension with an angiotensin-receptor blocker.
Julius S. et al. *N Engl J Med.* 2006; 354:1685-97

TROPHY Study

- **Objective:** The primary hypothesis of the study was to determine whether two years of treatment with candesartan reduces the incidence of hypertension two years after treatment and 2 years after discontinuation of treatment.



Characteristics of the Study Population

Table 1. Baseline Characteristics of the Study Participants.*

	Candesartan Group (N=391)	Placebo Group (N=381)
Age—yr	48.6±7.9	48.3±8.2
Male sex—no. (%)	231 (59.1)	229 (60.1)
Race—no. (%)†		
White	312 (79.8)	321 (84.3)
Black	48 (12.3)	31 (8.1)
Other	31 (7.9)	29 (7.6)
Weight—kg	89.0±17	88.8±17.7
Body-mass index‡	29.9±5.1	30.0±5.5
Blood pressure—mm Hg		
Measured at clinic visit with automated device§	133.9±4.3/84.8±3.8	134.1±4.2/84.8±4.1

Main Results of the Study

Table 2. Incident Hypertension and Incidence of Serious Adverse Events.*

	Candesartan Group (N=391)	Placebo Group (N=381)	P Value	Relative Risk (95% CI)
New-onset hypertension				
No. of participants in whom hypertension developed	208	240		
Hypertension at year 2 visit—%	13.6	40.4	<0.001†	0.34 (0.25–0.44)
Hypertension at year 4 visit—%	53.2	63.0	0.007†	0.84 (0.75–0.95)
Hypertension during study period			<0.001‡	0.58 (0.49–0.70)
Clinical criteria for end-point determination				
BP at three clinic visits, ≥140 mm Hg systolic, ≥90 mm Hg diastolic, or both—no. (%)	142 (36)	168 (44)	0.03†	0.82 (0.69–0.98)
BP at any clinic visit ≥160 mm Hg systolic, ≥100 mm Hg diastolic, or both—no. (%)	15 (3.8)	19 (5.0)	0.49†	0.77 (0.40–1.49)
BP requiring pharmacologic treatment—no. (%)	45 (12)	48 (13)	0.66†	0.91 (0.62–1.34)
BP at month 48 clinic visit ≥140 mm Hg systolic, ≥90 mm Hg diastolic, or both—no. (%)	6 (1.5)	5 (1.3)	>0.99†	1.17 (0.36–3.80)

Main Results of the Study

	Candesartan Group (N=391)	Placebo Group (N=381)	P Value	Relative Risk (95% CI)
New-onset hypertension				
No. of participants in whom hypertension developed	208	240		
Hypertension at year 2 visit — %	13.6	40.4	<0.001†	0.34 (0.25–0.44)
Hypertension at year 4 visit — %	53.2	63.0	0.007†	0.84 (0.75–0.95)

At 2 Years	Hypertension		Total
	Yes(row %)	No	
Candesartan	53(13.6%)	338	391
Placebo	154(40.4%)	227	381
Total	207	565	772

ARR at 2 years: $40.4 - 13.6 = 26.8\%$ RR at 2 years: $.136 / .404 = .34$

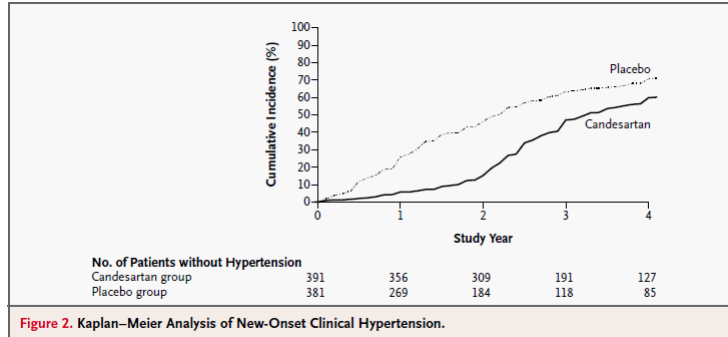
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	Candesartan Group (N=391)	Placebo Group (N=381)	P Value	Relative Risk (95% CI)
New-onset hypertension				
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Hypertension at year 4 visit — %	53.2	63.0	0.007†	0.84 (0.75–0.95)

At 4 Years	Hypertension		Total
	Yes(row %)	No	
Candesartan	208 (53.2%)	183	391
Placebo	240(63.0%)	141	381
Total	448	324	772

ARR at 4 years: $63.0 - 53.2 = 9.8\%$ RR at 4 years: $53.2 / 63.0 = .84$

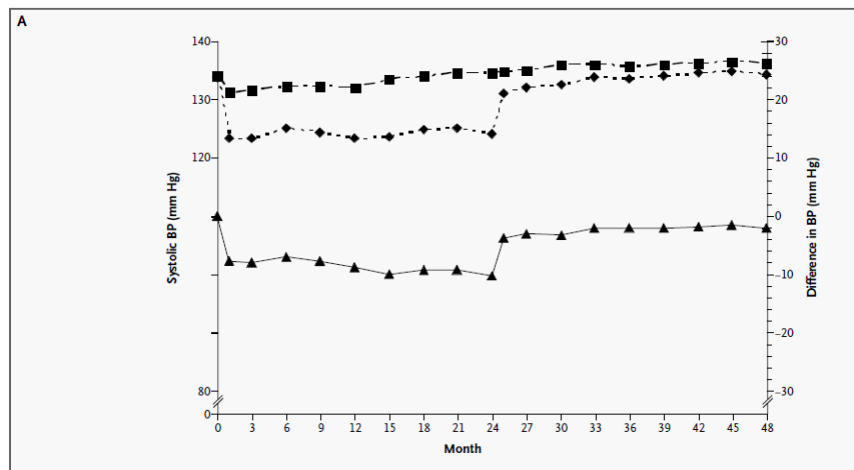
Cumulative Incidence of HT by Treatment Group



Kaplan-Meier Analysis shows if the overall cumulative incidence of HT is different between groups over time. It gives the full picture on the development of HT over the 4 year follow-up.

Note: Cumulative incidence is calculated as 100% - K-M curve

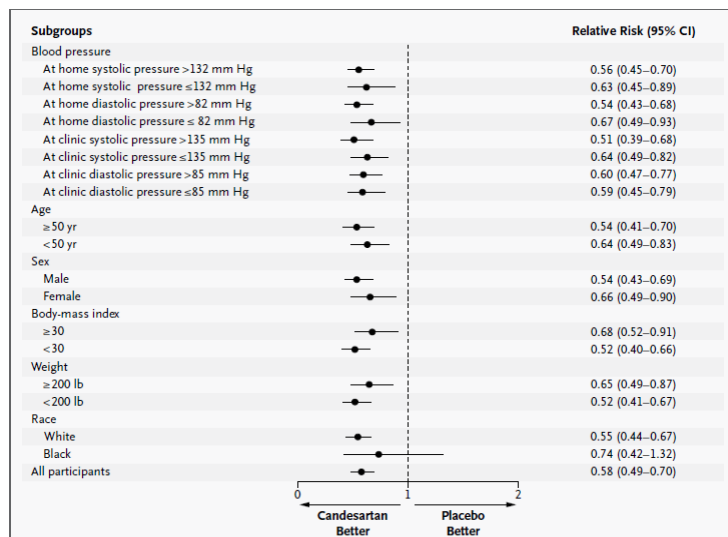
SBP Values Over 4 Years



SBP curve (mean of SBP at each visit) over 4 years

Two-sample t-test: Showed 2.0 mm Hg ($p=0.037$) decrease in SBP at year 4 due to Candesartan

Subgroup Analysis: Does Candesartan work the same way for different subgroups



Summary Points

Tests for Comparing Proportions: $H_0: p_1 = p_2$ vs. $H_A: p_1 \neq p_2$

Statistical test

- Two-sample normal theory test

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

- Chi-square test
 - Use χ^2_k where $k=(nrow-1) \times (ncol-1)$

- Fisher's exact test
 - Calculates the exact p-value

Used when

$$n_1\hat{p}_1(1-\hat{p}_1) > 5$$

$$n_2\hat{p}_2(1-\hat{p}_2) > 5$$

$n > 5$ in all cells

n is small and the other two tests does not apply

Summary Points

Measure of association (treatment effect) for Dichotomous Outcomes.

“Risk” is defined as: $Pr(Y=Yes)=p$, (p_1 is for treatment, p_2 is for control)

<u>Measure of association</u>	<u>Interpretation</u>
<ul style="list-style-type: none"> • Absolute Risk Reduction (ARR) <ul style="list-style-type: none"> – $ARR = p_2 - p_1$ 	(ARR=0 do not reject $H_0: p_1 = p_2$)
<ul style="list-style-type: none"> • Relative Risk (RR) <ul style="list-style-type: none"> – $RR = \frac{p_1}{p_2}$ 	(RR=1 do not reject $H_0: p_1 = p_2$)
<ul style="list-style-type: none"> • Odds Ratio (OR) <ul style="list-style-type: none"> – $ODDs = \frac{Pr(Y=1)}{Pr(Y=0)} = \frac{p}{1-p}$ – $OR = \frac{ODDs(Trt)}{ODDs(Control)} = \frac{p_1/(1-p_1)}{p_2/(1-p_2)}$ 	(OR=1 do not reject $H_0: p_1 = p_2$)