



**BIMM 143**  
**Unsupervised Learning II**  
Lecture 9  
**Barry Grant**  
UC San Diego  
<http://thegrantlab.org/bimm143>

## Recap of Lecture 8

- Introduction to machine learning
  - Unsupervised, supervised and reinforcement learning
- Clustering
  - K-means clustering
  - Hierarchical clustering
- Dimensionality reduction, visualization and 'structure' analysis
  - Principal Component Analysis (PCA)

[Muddy Point Feedback Link] :-)

## Recap: **PCA objectives**

- To reduce dimensionality
- To visualize multidimensional data
- To choose the most useful variables (features)
- To identify groupings of objects (e.g. genes/samples)
- To identify outliers

A long time ago in a galaxy far,  
far away....



David Robinson  
@drob

Following

Every linear algebra class

Me: What are eigenvectors

Teacher: You can think of them as an n-dimensional kernel subspace

Me: No I can't

3:08 PM - 28 Mar 2016

702 Retweets 1,384 Likes



# PRINCIPAL COMPONENTS

Principal components are the linear combination of features that have the maximum variance out of all linear combinations.

Alternative interpretation: Principal components are low dimensional linear surfaces closest to the observations.

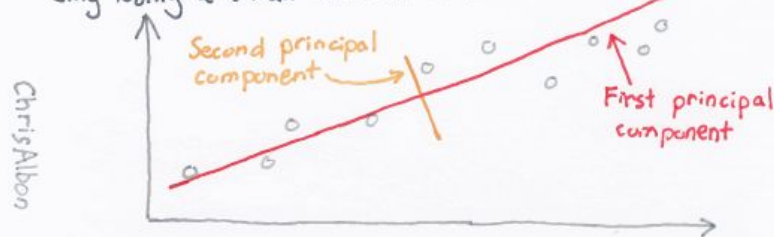
ChrisAlbon

Reference: @chrisalbon

# PCA

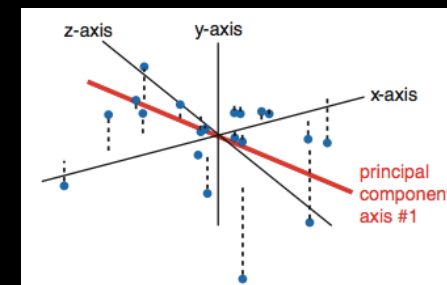
## PRINCIPAL COMPONENT ANALYSIS

PCA projects the features onto the principal components. The motivation is to reduce the features dimensionality while only losing a small amount of information.



Reference: @chrisalbon

# PCA



- The first principal component (PC) follows a “best fit” through the data points. Other PCs must cross the origin of the plot, and must be orthogonal.

# Practical PCA issue: Scaling

```
> data(mtcars)
> head(mtcars)
```

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

```
# Means and standard deviations vary a lot
> round(colMeans(mtcars), 2)
```

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
	20.09	6.19	230.72	146.69	3.60	3.22	17.85	0.44	0.41	3.69	2.81

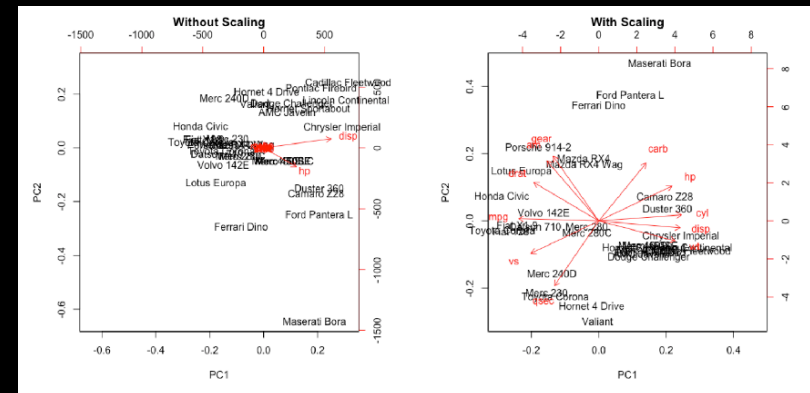
```
> round(apply(mtcars, 2, sd), 2)
```

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
	6.03	1.79	123.94	68.56	0.53	0.98	1.79	0.50	0.50	0.74	1.62

# Practical PCA issue: Scaling

prcomp(x, scale=FALSE)

prcomp(x, scale=TRUE)



## Your turn!

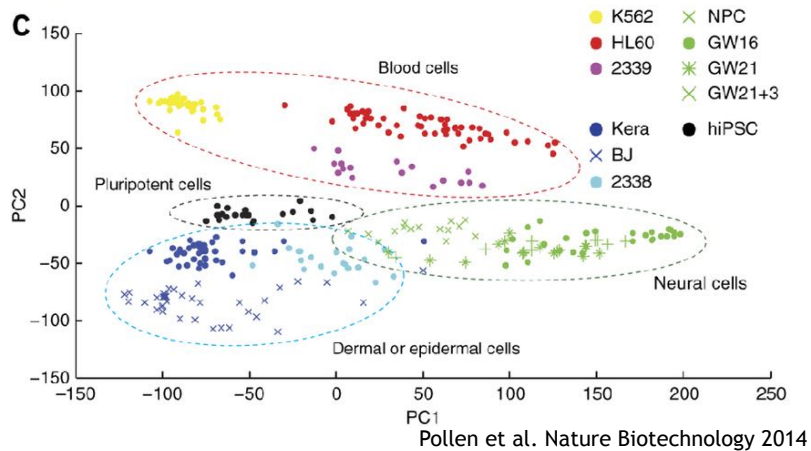
[Unsupervised Learning Mini-Project](#)

**Input:** read, View/head,  
**PCA:** prcomp,  
**Cluster:** kmeans, hclust  
**Compare:** plot, table, etc.

## Reference Slides

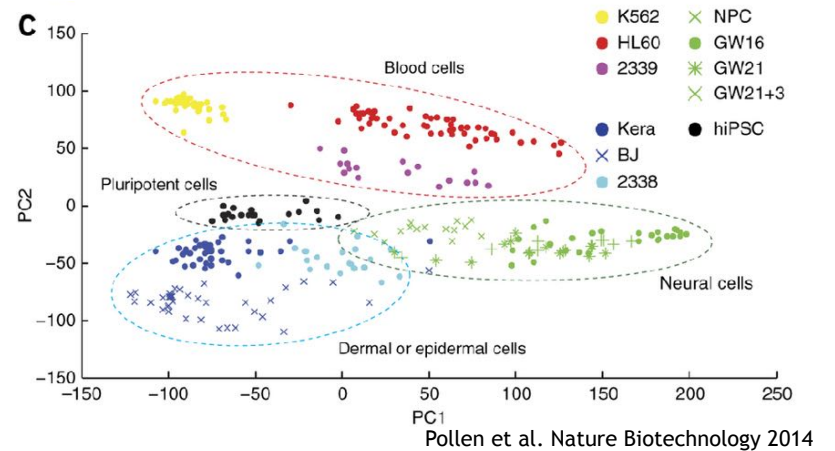
## This PCA plot shows clusters of cell types.

This graph was drawn from single-cell RNA-seq.  
There were about 10,000 transcribed genes in each cell.



## This PCA plot shows clusters of cell types.

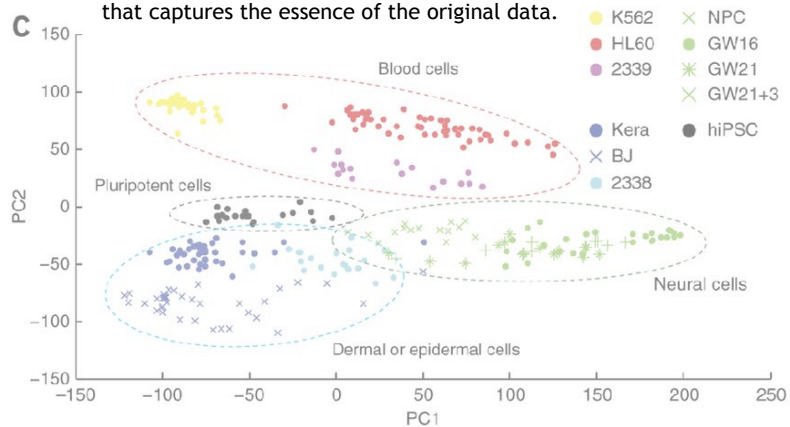
Each dot represents a single-cell and its transcription profile.  
The general idea is that cells with similar transcription should cluster.



## This PCA plot shows clusters of cell types.

How does transcription from 10,000 genes get compressed to a single dot on a graph?

PCA is a method for compressing a lot of data into something that captures the essence of the original data.



## What does PCA aim to do?

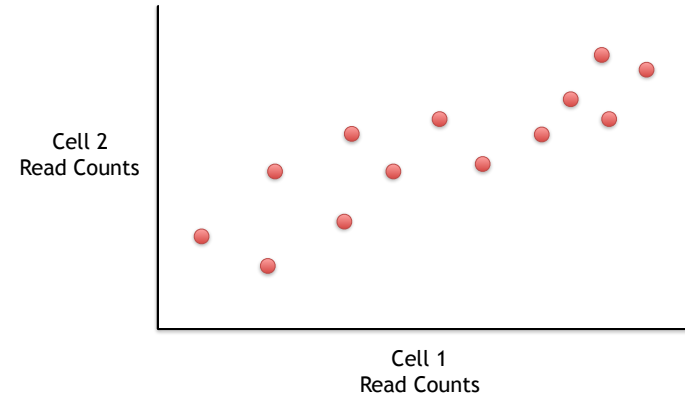
- PCA takes a dataset with a lot of dimensions (i.e. lots of cells) and flattens it to 2 or 3 dimensions so we can look at it.
  - It tries to find a meaningful way to flatten the data by focusing on the things that are different between cells. (much, much more on this later)
- This is sort of like flattening a Z-stack of microscope images to make a single 2-D image for publication.

# A PCA example

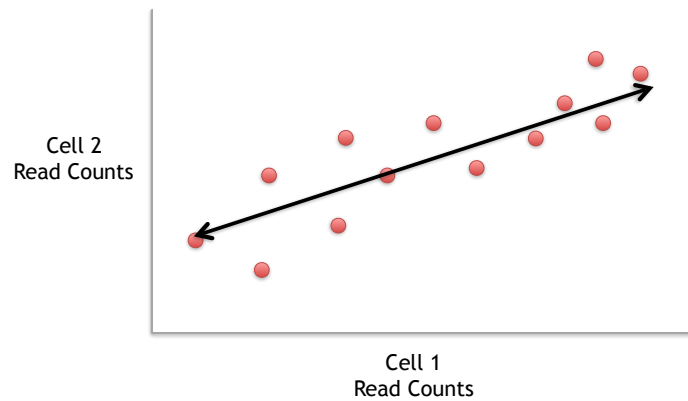
Again, we'll start with just two cells  
Here's the data:

Gene	Cell1 reads	Cell2 reads
a	10	8
b	0	2
c	14	10
d	33	45
e	50	42
f	80	72
g	95	90
h	44	50
i	60	50
... (etc)	... (etc)	... (etc)

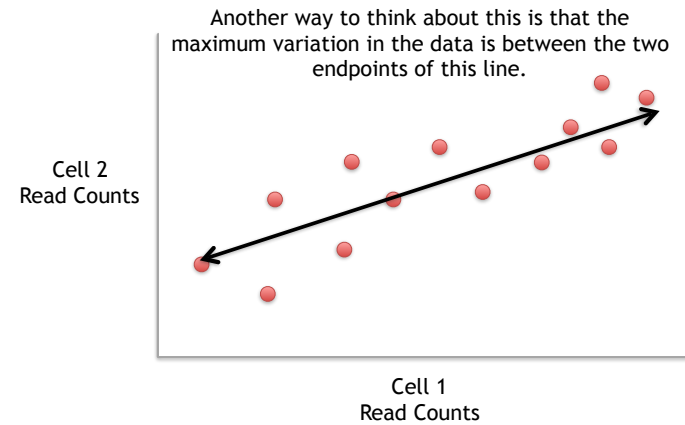
Here is a 2-D plot of the data from 2 cells.



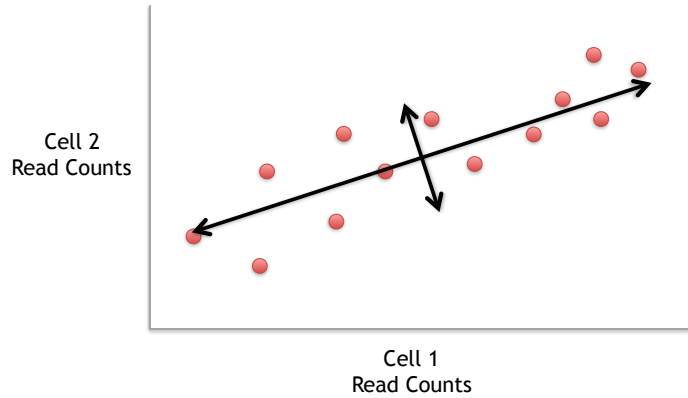
Generally speaking, the dots are spread out along a diagonal line.



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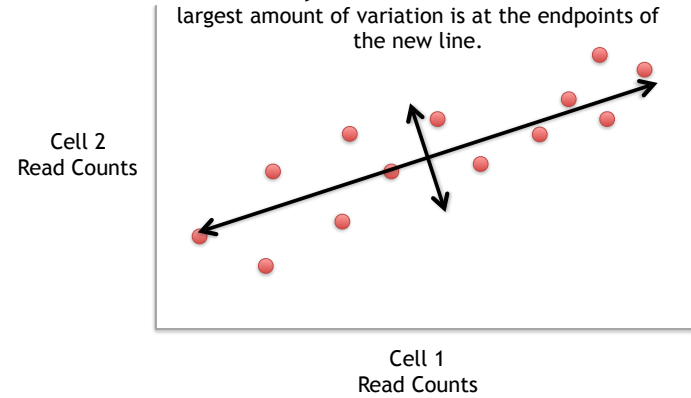


Generally speaking, the dots are also spread out a little above and below the first line.

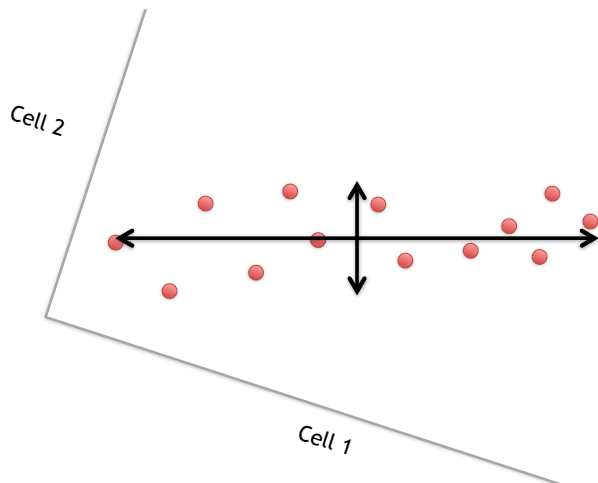


Generally speaking, the dots are also spread out a little above and below the first line.

Another way to think about this is that the 2<sup>nd</sup> largest amount of variation is at the endpoints of the new line.

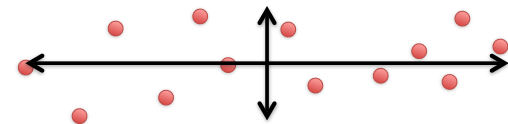


If we rotate the whole graph, the two lines that we drew make new X and Y axes.



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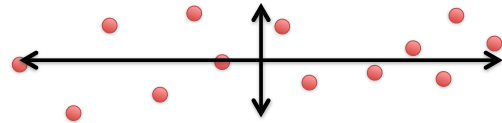
This makes the left/right, above/below variation easier to see.



If we rotate the whole graph, the two lines that we drew make new X and Y axes.

This makes the left/right, above/below variation easier to see.

1) The data varies a lot left and right



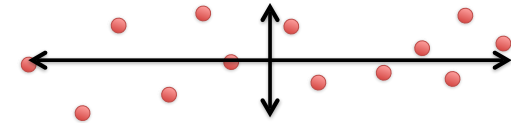
If we rotate the whole graph, the two lines that we drew make new X and Y axes.

This makes the left/right, above/below variation easier to see.

1) The data varies a lot left and right



2) The data varies a little up and down



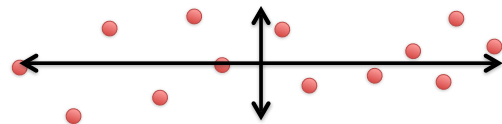
If we rotate the whole graph, the two lines that we drew make new X and Y axes.

This makes the left/right, above/below variation easier to see.

1) The data varies a lot left and right



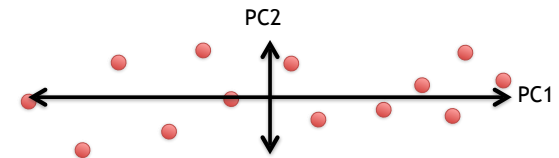
2) The data varies a little up and down



Note: All of the points can be drawn in terms of left/right + up/down, just like any other 2-D graph.

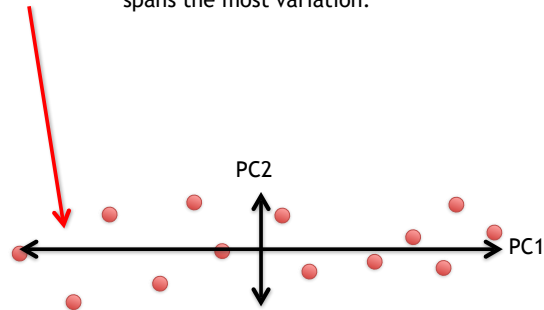
That is to say, we do not need another line to describe “diagonal” variation - we’ve already captured the two directions that can have variation.

These two “new” (or “rotated”) axes that describe the variation in the data are “Principal Components” (PCs)



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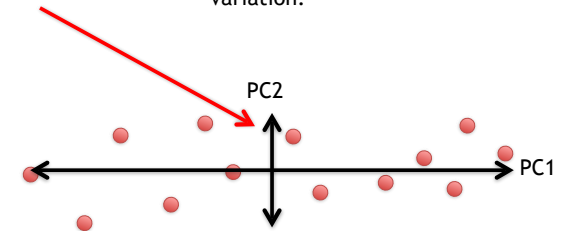
PC1 (the first principal component) is the axis that spans the most variation.



These two “new” axes that describe the variation in the data are “Principal Components” (PCs)

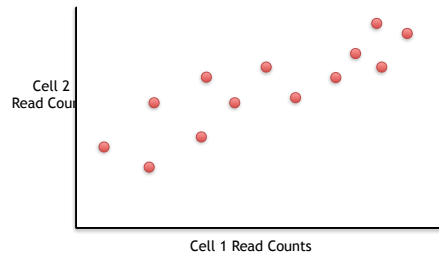
PC1 (the first principal component) is the axis that spans the most variation.

PC2 is the axis that spans the second most variation.



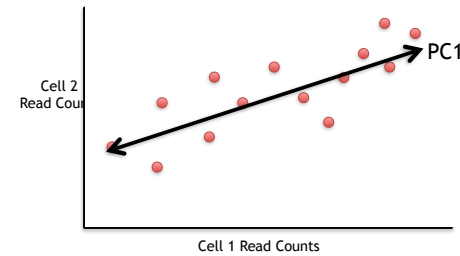
## General ideas so far...

- For each gene, we plotted a point based on how many reads were from each cell.



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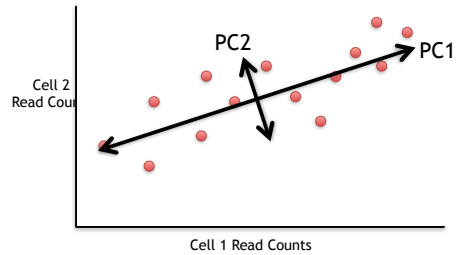


- PC1 captures the direction where most of the variation is.



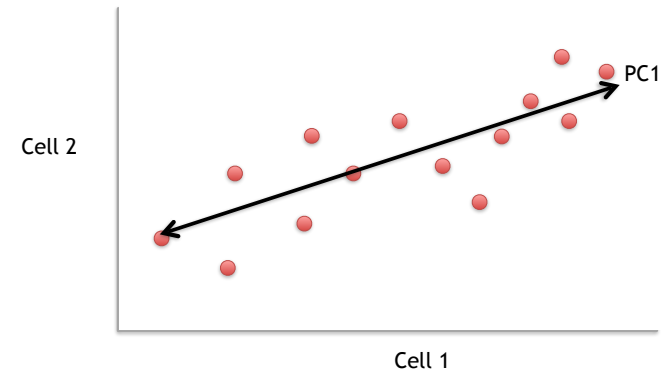
## General ideas so far...

- For each gene, we plotted a point based on how many reads were from each cell.

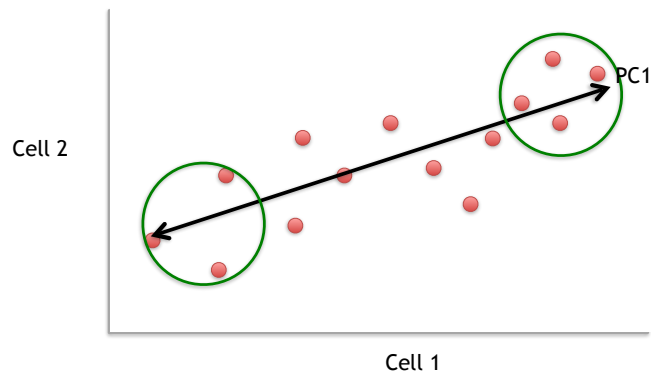


- PC1 captures the direction where most of the variation is.
- PC2 captures the direction with the 2<sup>nd</sup> most variation.

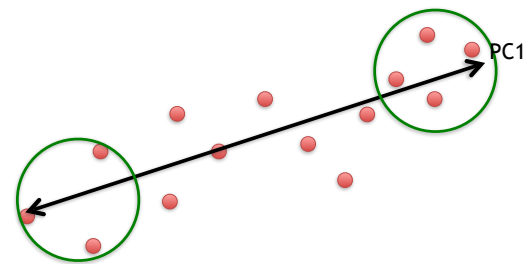
For now, let's focus on PC1



The length and direction of PC1 is mostly determined by the circled genes.

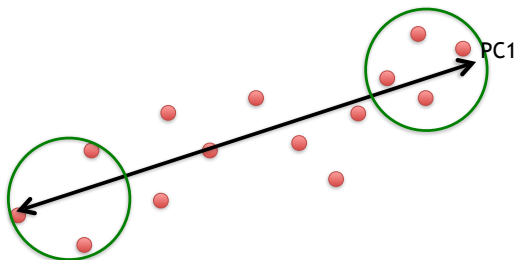


The length and direction of PC1 is mostly determined by the circled genes.



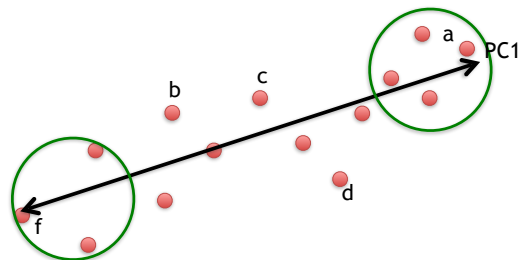
The length and direction of PC1 is mostly determined by the circled genes.

We can score genes based on how much they influence PC1.



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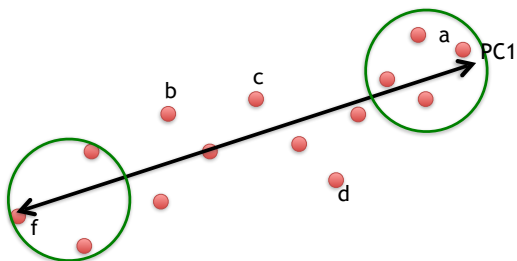
We can score genes based on how much they influence PC1.



Gene	Influence on PC1
a	high
b	low
c	low
d	low
e	high
f	high
...	...

The length and direction of PC1 is mostly determined by the circled genes.

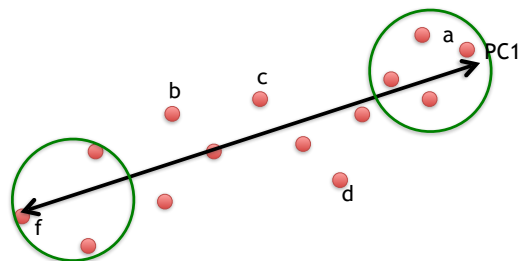
Some genes have more influence on PC1 than others.



Gene	Influence on PC1
a	high
b	low
c	low
d	low
e	high
f	high
...	...

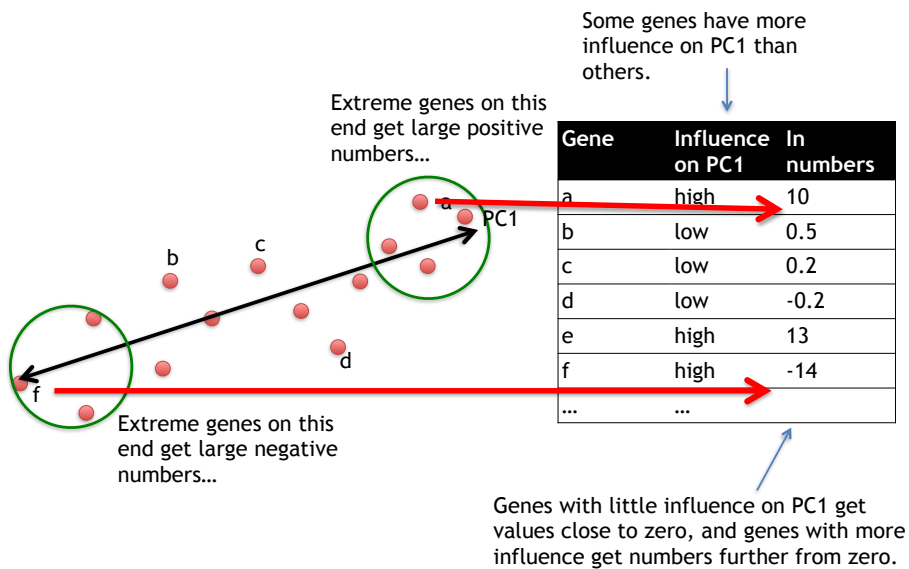
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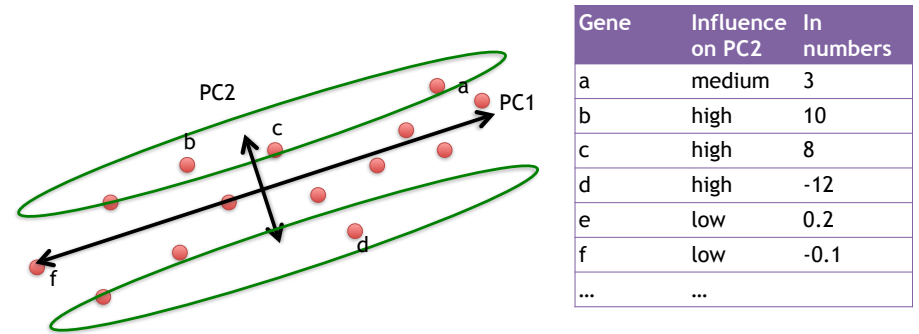


Gene	Influence on PC1	In numbers
a	high	10
b	low	0.5
c	low	0.2
d	low	-0.2
e	high	13
f	high	-14
...	...	...

Genes with little influence on PC1 get values close to zero, and genes with more influence get numbers further from zero.



## Genes that influence PC2



## Our two PCs

PC1			PC2		
Gene	Influence on PC1	In numbers	Gene	Influence on PC2	In numbers
a	high	10	a	medium	3
b	low	0.5	b	high	10
c	low	0.2	c	high	8
d	low	-0.2	d	high	-12
e	high	13	e	low	0.2
f	high	-14	f	low	-0.1
...	...	...	...	...	...

## Using the two Principal Components to plot cells

Combining the read counts for all genes in a cell to get a single value.

PC1			PC2		
Gene	Influence on PC1	In numbers	Gene	Influence on PC2	In numbers
a	high	10	a	medium	3
b	low	0.5	b	high	10
c	low	0.2	c	high	8
d	low	-0.2	d	high	-12
e	high	13	e	low	0.2
f	high	-14	f	low	-0.1
...	...	...	...	...	...

## Using the two Principal Components to plot cells

Combining the read counts for all genes in a cell to get a single value.

The original read counts

Gene	Cell1	Cell2
a	10	8
b	0	2
c	14	10
d	33	45
e	50	42
f	80	72
g	95	90
h	44	50
i	60	50
etc	etc	etc

PC1

Gene	Influence on PC1	In numbers
a	high	10
b	low	0.5
c	low	0.2
d	low	-0.2
e	high	13
f	high	-14
...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

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a	high	10
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...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

Cell1 PC1 score = (read count \* influence) + ... for all genes

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Combining the read counts for all genes in a cell to get a single value.

The original read counts

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a	10	8
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...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

Cell1 PC1 score = (10 \* 10) + ...

## Using the two Principal Components to plot cells

Combining the read counts for all genes in a cell to get a single value.

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...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
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c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

Cell1 PC1 score = (10 \* 10) + (0 \* 0.5) + ...

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Combining the read counts for all genes in a cell to get a single value.

The original read counts

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a	10	8
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e	high	13
f	high	-14
...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

$$\text{Cell1 PC1 score} = (10 * 10) + (0 * 0.5) + \dots \text{etc...} = 12$$

## Using the two Principal Components to plot cells

Combining the read counts for all genes in a cell to get a single value.

The original read counts

Gene	Cell1	Cell2
a	10	8
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f	80	72
g	95	90
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etc	etc	etc

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a	high	10
b	low	0.5
c	low	0.2
d	low	-0.2
e	high	13
f	high	-14
...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

$$\text{Cell1 PC1 score} = (10 * 10) + (0 * 0.5) + \dots \text{etc...} = 12$$

$$\text{Cell1 PC2 score} = (10 * 3) + \dots$$

## Using the two Principal Components to plot cells

Combining the read counts for all genes in a cell to get a single value.

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a	10	8
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a	high	10
b	low	0.5
c	low	0.2
d	low	-0.2
e	high	13
f	high	-14
...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

$$\text{Cell1 PC1 score} = (10 * 10) + (0 * 0.5) + \dots \text{etc...} = 12$$

$$\text{Cell1 PC2 score} = (10 * 3) + (0 * 10) + \dots$$

## Using the two Principal Components to plot cells

Combining the read counts for all genes in a cell to get a single value.

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Gene	Cell1	Cell2
a	10	8
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d	33	45
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PC1

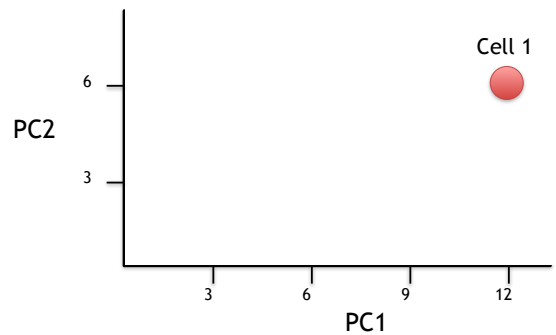
Gene	Influence on PC1	In numbers
a	high	10
b	low	0.5
c	low	0.2
d	low	-0.2
e	high	13
f	high	-14
...	...	...

PC2

Gene	Influence on PC2	In numbers
a	medium	3
b	high	10
c	high	8
d	high	-12
e	low	0.2
f	low	-0.1
...	...	...

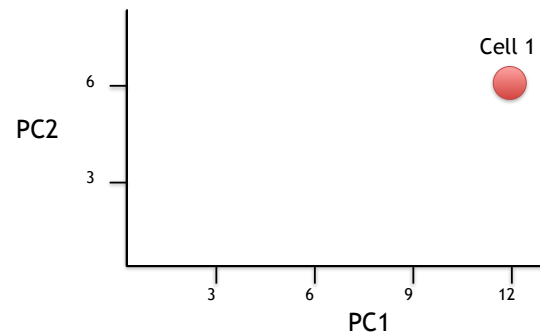
$$\text{Cell1 PC1 score} = (10 * 10) + (0 * 0.5) + \dots \text{etc...} = 12$$

$$\text{Cell1 PC2 score} = (10 * 3) + (0 * 10) + \dots \text{etc...} = 6$$

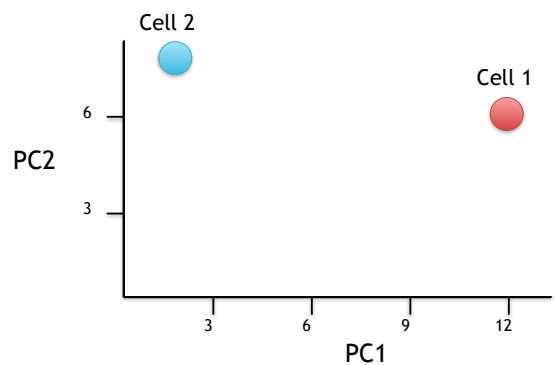


Cell1 PC1 score =  $(10 * 10) + (0 * 0.5) + \dots \text{etc} \dots = 12$

Cell1 PC2 score =  $(10 * 3) + (0 * 10) + \dots \text{etc} \dots = 6$



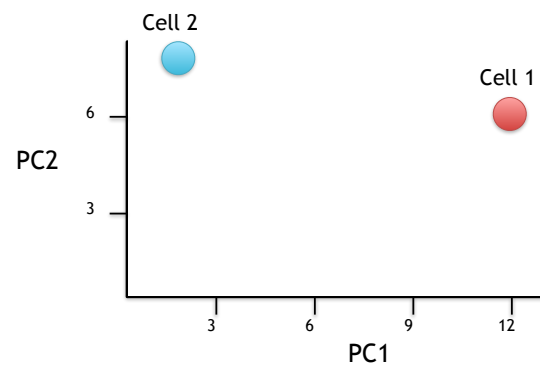
Now calculate scores for Cell2



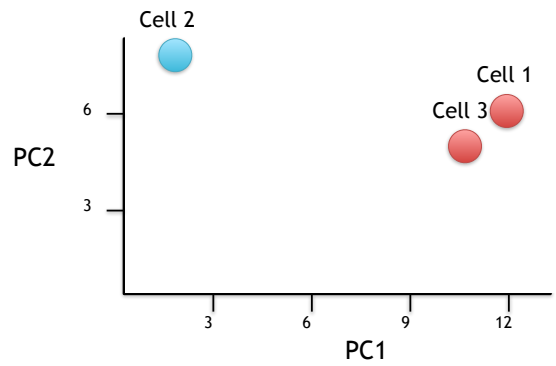
Now calculate scores for Cell2

Cell2 PC1 score =  $(8 * 10) + (2 * 0.5) + \dots \text{etc} \dots = 2$

Cell2 PC2 score =  $(8 * 3) + (2 * 10) + \dots \text{etc} \dots = 8$



If we sequenced a third cell, and its transcription was similar to cell 1, it would get scores similar to cell 1's.



If we sequenced a third cell, and its transcription was similar to cell 1, it would get scores similar to cell 1's.

Hooray! We know how they plotted all of the cells!!!

